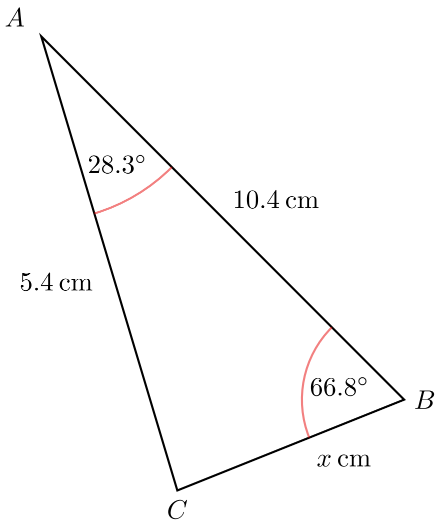
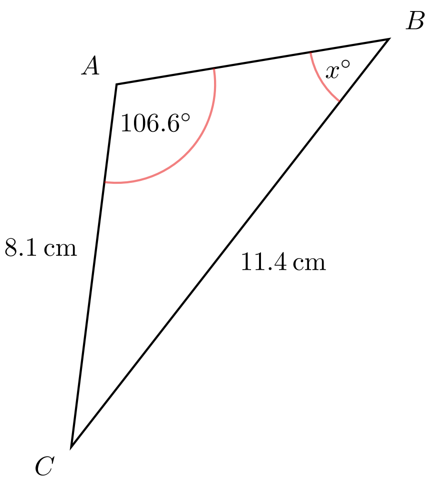
Solution Sheet

1 By the sine rule,



$$\frac{x}{\sin 28.3} = \frac{10.4}{\sin 66.8}$$
$$x = \frac{10.4 \sin 28.3}{\sin 66.8}$$
$$x = \frac{4.93}{0.919}$$
$$x = 5.36$$

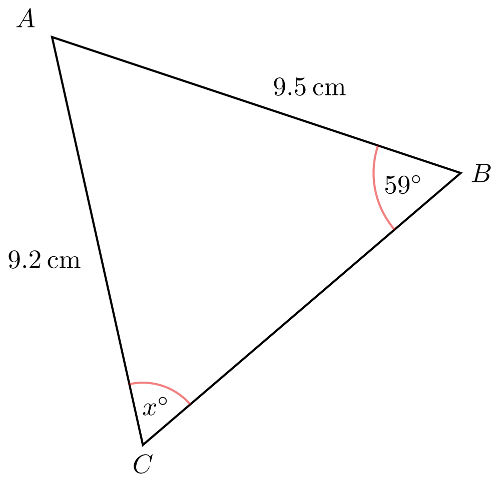
2 By the sine rule,



$$\frac{\sin x}{8.1} = \frac{\sin 106.6}{11.4}$$
$$\sin x = \frac{8.1 \sin 106.6}{11.4}$$
$$\sin x = \frac{7.76}{11.4}$$
$$\sin x = 0.681$$
$$x = \sin^{-1}(0.681)$$
$$x = 42.9$$

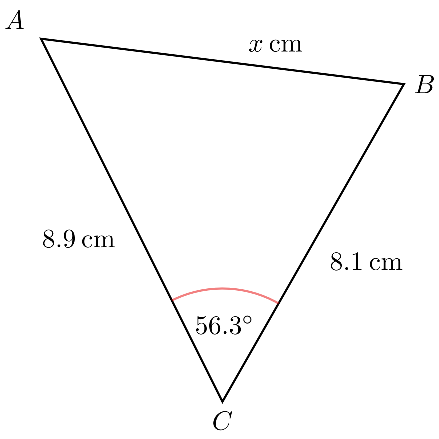
3 By the sine rule,

$$\frac{\sin x}{9.5} = \frac{\sin 59}{9.2}$$
$$\sin x = \frac{9.5 \sin 59}{9.2}$$
$$\sin x = \frac{8.14}{9.2}$$
$$\sin x = 0.885$$



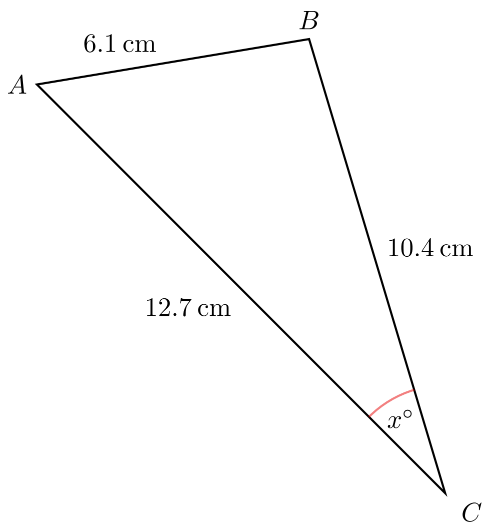
$$x = \sin^{-1}(0.885)$$
$$x = 62.3$$
$$x = 180 - 62.3$$
$$= 118$$

4 By the cosine rule,



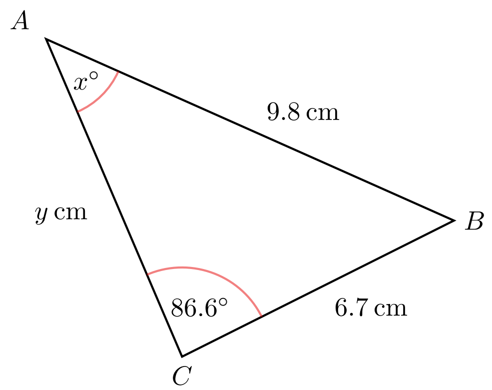
$$x^2 = 8.1^2 + 8.9^2 - 2 \times 8.1 \times 8.9 \cos 56.3$$
$$x^2 = 145 - 144 \cos 56.3$$
$$x^2 = 64.8$$
$$x = \sqrt{64.8}$$
$$x = 8.05$$

5 By the cosine rule,



$$\cos x = \frac{10.4^2 + 12.7^2 - 6.1^2}{2 \times 10.4 \times 12.7}$$
$$\cos x = \frac{232}{264}$$
$$\cos x = 0.879$$
$$x = \cos^{-1}(0.879)$$
$$x = 28.5$$

6



By the sine rule,

$$\begin{aligned}\frac{\sin x}{6.7} &= \frac{\sin 86.6}{9.8}\\ \sin x &= \frac{6.7\sin 86.6}{9.8}\\ \sin x &= \frac{6.69}{9.8}\\ \sin x &= 0.682\\ x &= \sin^{-1}(0.682)\\ x &= 43.0\end{aligned}$$

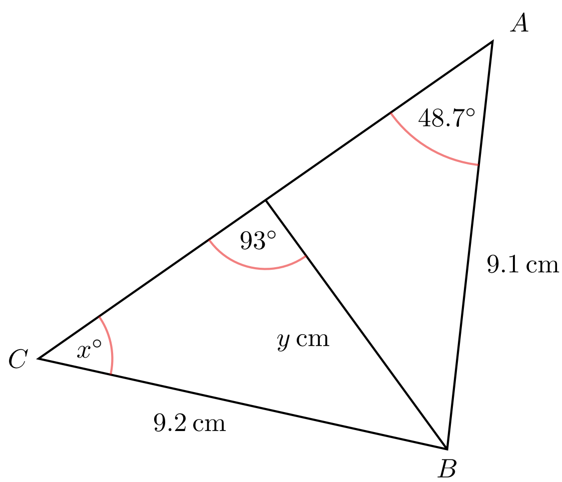
Therefore, since the internal angles of a triangle add up to 180,

$$\begin{aligned}x &= 180 - 86.6 - 43.0\\ &= 50.4\end{aligned}$$

Finally, by the sine rule,

$$\begin{aligned}\frac{y}{\sin\angle ABC} &= \frac{9.8}{\sin 86.6}\\ \frac{y}{\sin 50.4} &= \frac{9.8}{\sin 86.6}\\ y &= \frac{9.8\sin 50.4}{\sin 86.6}\\ y &= \frac{7.55}{0.998}\\ y &= 7.56\end{aligned}$$

7 By the sine rule,



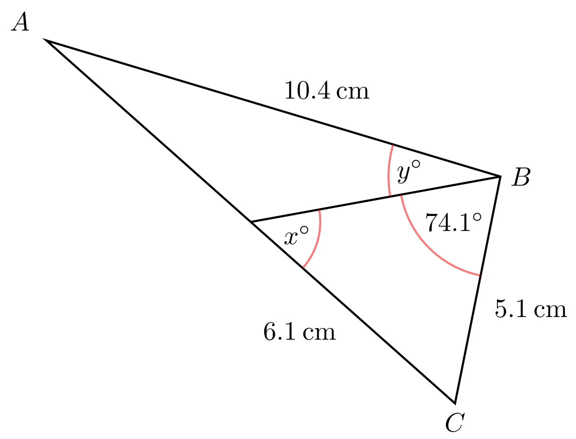
$$\begin{aligned}\frac{\sin x}{9.1} &= \frac{\sin 48.7}{9.2}\\ \sin x &= \frac{9.1\sin 48.7}{9.2}\end{aligned}$$

$$\begin{aligned}\frac{y}{\sin 48} &= \frac{9.2}{\sin 93}\\ y &= \frac{9.2\sin 48}{\sin 93}\end{aligned}$$

$$\sin x = \frac{6.84}{9.2}$$
$$\sin x = 0.743$$
$$x = \sin^{-1}(0.743)$$
$$x = 48$$

$$y = \frac{6.84}{0.999}$$
$$y = 6.85$$

8 By the sine rule,



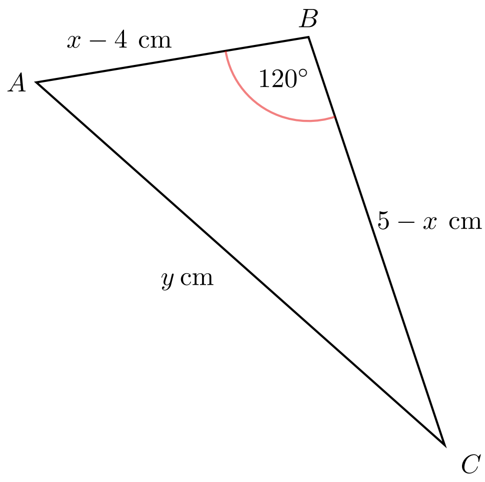
$$\frac{\sin x}{5.1} = \frac{\sin 74.1}{6.1}$$
$$\sin x = \frac{5.1 \sin 74.1}{6.1}$$
$$\sin x = \frac{0.804}{6.1}$$
$$\sin x = 0.804$$
$$x = \sin^{-1}(0.804)$$
$$x = 53.5$$

$$\angle ACB = 180 - 74.1 - 53.5 = 52.4$$

$$\frac{\sin \angle BAC}{5.1} = \frac{\sin 52.4}{10.4}$$
$$\sin \angle BAC = \frac{5.1 \sin 52.4}{10.4}$$
$$\sin \angle BAC = \frac{4.04}{10.4}$$
$$\sin \angle BAC = 0.389$$
$$\angle BAC = \sin^{-1}(0.389)$$
$$\angle BAC = 22.9$$

$$y = 180 - 22.9 - (180 - 53.5) = 30.6$$

9 By the cosine rule



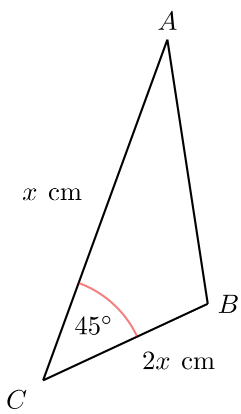
$$
\begin{aligned}
y^2 &= (x-4)^2 + (5-x)^2 - 2(x-4)(5-x)\cos 120 \\
&= (x-4)^2 + (5-x)^2 + (x-4)(5-x) \\
&= \left(x^2 - 8x + 16\right) + \left(25 - 10x + x^2\right) + \left(9x - x^2 - 20\right) \\
&= x^2 - 9x + 21
\end{aligned}
$$

$$
\begin{aligned}
y^2 &= x^2 - 9x + 21 \\
&= \left(x - \frac{9}{2}\right)^2 + 21 - \frac{81}{4} \\
&= \left(x - \frac{9}{2}\right)^2 + \frac{3}{4}
\end{aligned}
$$

The minimum value of y^2 value occurs when $(x - \frac{9}{2})^2 = 0$, therefore $x = \frac{9}{2}$.

Hence the minimum value of y^2 is $\frac{3}{4}$.

10 By the sine area rule,



$$
\begin{aligned}
\text{Area} &= \frac{1}{2}(x)(2x)\sin 45 \\
8\sqrt{2} &= \frac{1}{2}\left(2x^2\right) \times \frac{1}{\sqrt{2}} \\
32 &= 2x^2 \\
32 &= 2x^2 \\
0 &= 2x^2 - 32 \\
0 &= 2\left(x^2 - 16\right) \\
0 &= 2\left(x^2 - 4^2\right) \\
0 &= 2(x+4)(x-4) \\
x &= -4,\ 4
\end{aligned}
$$

$$x = 4$$